Math 342: Project 2

Connor Emmons

Documentation: The main Project 2 MatLab script and all required dependencies are located in the Project 2 folder found here: <https://github.com/Connor-Lemons/Emmons-Math-342>. No other resources used.

Problem 1: Arrange the parameters of RK4 in a Butcher Tableu

(1)

(2)

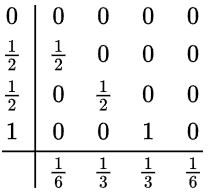
(3)

(4)

(5)

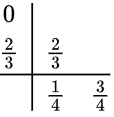
(6)

The Butcher Tableu representation of this method is:

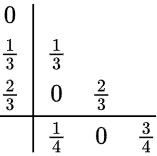


Problem 2: For the three RK methods shown below, given the Butcher Tableau representation, give the equations which define those methods.

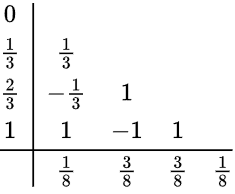
Ralston Method



Heun Third-Order Method



Runge-Kutta 3/8 Rule



The equations for the Ralston Method are:

(1)

(2)

(3)

(4)

The equations for the Heun Third-Order Method are:

(5)

(6)

(7)

(8)

(9)

The equations for the Runga-Kutta 3/8 Rule are:

(10)

(11)

(12)

(13)

(14)

(15)

Problem 3: Implement the three methods discussed in Problem 2 in MatLab.

Code can be found in the GitHub page or attached at the end of this document.

<https://github.com/Connor-Lemons/Emmons-Math-342/tree/main/Project%202>

Problem 4: Implement the Runge-Kutta-Fehlberg method according to Algorithm 5.3.

Code can be found in the GitHub page or attached at the end of this document.

<https://github.com/Connor-Lemons/Emmons-Math-342/tree/main/Project%202>

Problem 5: Rewrite the equation , where is the number of susceptible (non-infective) individuals and y(t) is the number of infective individuals, as a function solely of and , which represents the total population.

Note that the the sum of the number of non-infective individuals and the number of infective individuals is the size of the population. This gives:

(1)

Solving for gives:

(2)

Making this substitution in the original equation gives:

(3)

(4)

Problem 6: Use the methods described in Problems 1-4 to obtain various estimated solutions to equation (4) in Problem 5.

Parameters:

(1)

(2)

(3)

(4)

(5)

The results of these methods can be found in the Github page or attached at the end of this document.

<https://github.com/Connor-Lemons/Emmons-Math-342/tree/main/Project%202>

The final estimation of the number of infective individuals from each method are shown below. Note that all values are rounded to the nearest integer.

:

Ralston:

Heun:

RK38:

RK4:

:

Ralston:

Heun:

RK38:

RK4:

RKF45:

All estimations are relatively similar. The higher the order of the method, the more accurate it is likely to be. Running the methods with a smaller also likely produced more accurate results, with the most accurate method being the RKF45 method due to its high order and optimizing of to match a specified tolerance level.